The Scattering Parameters and Directional Coupler Analysis of Characteristically Terminated Asymmetric Coupled Transmission Lines in an Inhomogeneous Medium

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Abstract —The scattering matrix of asymmetric coupled two-line structures in an inhomogeneous medium terminated in a set of impedances which are equal to the characteristic impedances of the individual, uncoupled lines has been derived in terms of the coupled-mode parameters. It has been proved that the structure can compose an ideal, backward-coupling directional coupler, perfectly matched and isolated at all frequencies, if the inductive k_L and capacitive k_C coupling coefficients of the coupled lines are equal. The effect of the nonideal equalization of the coupling coefficients on the coupler critical parameters is then investigated. The normal-mode parameters (mode numbers and mode impedances) in the proximity of the point when $k_L = k_C$ and at that point are also examined. Numerical results confirm the validity of the developed analysis and prove the possibility of a very high directivity asymmetrical coupler design.

I. Introduction

ASYMMETRIC coupled lines seem to be of less practical importance in microwave passive circuits than their symmetric counterparts. The main reason is that directional coupling occurs most distinctly and in a more useful practical form when the coupled lines are identical. Furthermore, in contrast to the symmetric arrangement, it is much more complicated to calculate parameters of asymmetric coupled lines and to analyze and design passive circuits with these lines.

Nevertheless, recently [1]–[9] a comprehensive study of the physical behavior of guided modes and properties of asymmetric coupled lines in an inhomogeneous medium has been made, and several papers [10]–[16] dealing with their analysis and applications have been published. Growth of interest is due to the possibility of achieving tightly coupled sections of transmission lines in certain structures which are electrically nonsymmetrical, although they can be structurally symmetrical [13], [14]. Using an asymmetrical design of forward-coupling microstrip hybrids [16], a much higher directivity and a considerably wider bandwidth can also be achieved.

Among passive circuits where asymmetric coupled lines can be applied, directional couplers constitute an important class of devices commonly used in microwave techniques. In 1966 Cristal [17] was the first to introduce coupled lines of unequal characteristic impedances in designing asymmetrical couplers, offering the potential to realize simultaneously in a single device directional coupling and impedance transformation. Cristal's method of designing an ideal coupler, perfectly matched and isolated at all frequencies, is valid only in the case of homogeneous TEM lines. However, it is still used in designing inhomogeneous coupled-line couplers [14], because of a lack of an exact rigorous design approach. The approximate TEM approach can give reasonably good responses in all cases when the relative difference between phase velocities of c and π independent normal modes propagating in a system of nonhomogeneous asymmetric coupled lines is small enough. Then, one can accept that modal characteristic impedances of two lines, Z_{ic} and $Z_{i\pi}$, $i = 1,2$, are approximately equal to the even- and odd-mode impedances defined by Cristal for each individual line of a set of asymmetric coupled lines in a homogeneous medium, Z_{ie} and Z_{io}, respectively.

Gunton and Paige [11] have shown that for a general asymmetric coupled-line coupler, resistive terminations can be chosen so that the normal modes of the coupled system are reflected without conversion from one to the other. The introduction of such "non-mode-converting" (n.m.c.) terminations leads to relatively simple scattering S-parameter expressions describing the behavior of very general directional couplers. These S parameters are expressed in terms of the normal-mode parameters (voltage mode numbers R_c and R_π and mode impedances Z_{ic} and $Z_{i\pi}$). Moreover, the well known "backward" coupled-line couplers belong to that class of couplers so terminated. These couplers are ideal, provided they are matched and the

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normal-mode waves are propagating with the same velocities. This latter condition can be satisfied only in the case of the symmetrical lines. The conclusion is that the asymmetrical "backward" coupler with inhomogeneous coupled lines terminated by the n.m.c. impedances cannot be ideal; if it exists, their terminating impedances cause the conversion of the normal modes.

The authors of [14], [9], and [13] have observed that, for specific structural parameters of asymmetric coupled lines in an inhomogeneous medium, the impedances Z_{ic} and/or $Z_{i\pi}$ can assume values which are extremely high or extremely low (see, e.g., [14, table II]), and they can even be negative [9], [13]. Then, the modal impedances are not comparable to the ones defined by Cristal, and his approximate method of coupler design cannot be employed. This is the case that circuit designers do not know how to deal with [18]. The singular behavior of the normal-mode parameters of asymmetric coupled lines has not yet been clearly explained.

The purpose of this paper is to answer the question raised on the existence or nonexistence of an ideal asymmetric coupler and to present a method of design that can improve substantially the performances of couplers designed on the basis of existing methods. In Section II we will derive—applying a coupled-mode formulation of inhomogeneous lines successfully developed in the case of symmetrical lines [19]—explicit expressions for the S parameters of the asymmetrical structure. The terminating impedances are chosen in the most obvious and simple way; namely they are chosen to be equal to the characteristic impedances of the individual, uncoupled lines. It will be proved in this section that such a choice of terminations together with equalization of the coupling coefficients imposes conditions which are necessary and sufficient to realize the ideal coupler. The effect of a nonideal equalization of coupling coefficients on the return losses and coupler directivity will then be investigated. In Section III, the normal-mode parameters in proximity to the point where $k_L = k_C$ will be examined in detail, and simple new formulas useful in asymmetric coupled-line coupler design will be derived. Numerical results are given for specific applications.

II. Coupled-Mode Analysis

The first part of this section restates some of the derived results which are required and serve to introduce the notation. The second and third parts solve the problems raised above, in particular, deriving the S-parameter expressions for the structure of a generalized coupled-line coupler (shown in Fig. 1) for the two cases where $k_L = k_C$ and $k_L \simeq k_C$, respectively.

A. General Description

There are two alternative forms of the differential equations describing a lossless pair of coupled transmission lines: the first is written in terms of voltages V_1, V_2 and currents I_1, I_2 [2], [11], the second in terms of four well-known forward and backward power waves: a_+, b_+

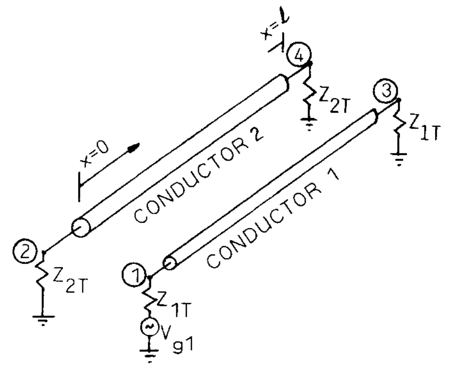

Fig. 1. Schematic representation of an asymmetric coupled-line coupler.

and a_-, b_-, respectively [19], [11]. These waves, which are treated as the coupled modes, are defined by

$$a_{\pm} = \frac{1}{2\sqrt{Z_1}}(V_1 \pm Z_1 I_1) \qquad b_{\pm} = \frac{1}{2\sqrt{Z_2}}(V_2 \pm Z_2 I_2)$$

where $Z_i = \sqrt{L_i/C_i}$, and L_i and C_i are the self-inductances and self-capacitances per unit length of line i in the presence of the other ($i = 1,2$).

In the literature, properties of asymmetric coupled lines are customarily described with their normal-mode parameters, defined by Tripathi [2] and obtained from solving the $V-I$ equations. From that analysis the Z matrix has been derived. There is the simple matrix relationship between the Z and S parameters:

$$(S) = [(Z)+(U)]^{-1}[(Z)-(U)] \tag{1}$$

where U is the unit matrix. This expression, unfortunately, brings very complicated, almost intractable formulas for the scattering parameters [1]. Therefore, we have chosen an alternative approach, in which the coupled-mode equations are solved together with the boundary conditions at the ports of the system.

Assuming a propagating wave term of the form $e^{j(\omega t - \beta x)}$ ($\omega = 2\pi f$, f the frequency, β the propagation constant) results in four normal modes of the coupler, two with positive eigenvalues and two with negative ones, corresponding to a pair of modes propagating in each direction. The eigenvalues are $\pm\,\beta_c$ and $\pm\,\beta_\pi$, where β_c and β_π are given [19] by

$$\beta_r = \beta_0\sqrt{1 \pm \delta} \qquad (r = c \text{ or } \pi). \tag{2a}$$

Here,

$$\beta_0 = \sqrt{\frac{\beta_1^2 + \beta_2^2}{2} - \beta_1\beta_2 k_L k_C}$$

$$\delta = \sqrt{1 - \frac{\beta_1^2\beta_2^2}{\beta_0^4}(1-k_L^2)(1-k_C^2)} \tag{2b}$$

$$\beta_i = \omega\sqrt{L_i C_i} \qquad k_L = L_m/\sqrt{L_1 L_2} \qquad k_C = C_m/\sqrt{C_1 C_2}$$

and L_m and C_m are the mutual inductance and mutual capacitance per unit length, respectively.

The eigenvector for the coupled-mode formulation can be easily found and written in two useful forms:

$$\begin{pmatrix} a_+ \\ b_+ \\ a_- \\ b_- \end{pmatrix}_r = \frac{1}{\delta_r}\begin{pmatrix} \delta_r \\ \alpha_{2,r} \\ \alpha_{3,r} \\ \alpha_{4,r} \end{pmatrix} \text{ and } \begin{pmatrix} a_+ \\ b_+ \\ a_- \\ b_- \end{pmatrix}_r = \frac{1}{v_r}\begin{pmatrix} \gamma_{1,r} \\ v_r \\ \gamma_{3,r} \\ \gamma_{4,r} \end{pmatrix} \tag{3}$$

where

$$\alpha_{2,r} = \gamma_{1,r} = \{(\beta_1+\beta_r)(\beta_2+\beta_r) + \beta_1\beta_2(s^2-d^2)\}\sqrt{\beta_1\beta_2}\,d$$

$$\alpha_{3,r}(\gamma_{4,r}) = \{2\sqrt{\beta_1\beta_2}\,\beta_{2(1)}\}\sqrt{\beta_1\beta_2}\,sd$$

$$\alpha_{4,r}(\gamma_{3,r}) = \{(\beta_{1(2)}+\beta_r)(\beta_{2(1)}-\beta_r) - \beta_1\beta_2(s^2-d^2)\}\sqrt{\beta_1\beta_2}\,s$$

$$\delta_r(v_r) = (\beta_1+\beta_r)(\beta_{2(1)}-\beta_r)(\beta_2+\beta_r) - \beta_1\beta_2(\beta_{2(1)}-\beta_r)d^2$$

$$s = \frac{1}{2}(k_C+k_L) \text{ and } d = \frac{1}{2}(k_C-k_L).$$

In this short notation the subscripts 1 or 2 in parentheses refer to the description of $\gamma_{i,r}$ and v_r, and the eigenvalues $\pm\beta_r$ are associated with the eigenvectors labeled $\pm r$.

B. The "Backward" Directional Coupler Analysis ($k_L = k_C = s, d = 0$)

The results reviewed in subsection A make it possible to investigate the behavior of coupled lines terminated so that they form a coupler with a voltage V_{g1} at its port 1, as is shown in Fig. 1. In general, the voltage generator V_{g1} and terminating impedances $Z_{1T} = Z_1$ and $Z_{2T} = Z_2$ will launch four normal modes, which means that each coupled mode a_+, b_+, a_-, and b_- can be regarded as a linear combination of launched modes:

$$\begin{pmatrix} a_+(x) \\ b_+(x) \\ a_-(x) \\ b_-(x) \end{pmatrix} = \begin{pmatrix} \lambda_{1,+c} \\ \lambda_{2,+c} \\ \lambda_{3,+c} \\ \lambda_{4,+c} \end{pmatrix} A_1 e^{-j\beta_c x} + \begin{pmatrix} \lambda_{1,-c} \\ \lambda_{2,-c} \\ \lambda_{3,-c} \\ \lambda_{4,-c} \end{pmatrix} A_2 e^{j\beta_c x} + \begin{pmatrix} \lambda_{1,+\pi} \\ \lambda_{2,+\pi} \\ \lambda_{3,+\pi} \\ \lambda_{4,+\pi} \end{pmatrix} A_3 e^{-j\beta_\pi x} + \begin{pmatrix} \lambda_{1,-\pi} \\ \lambda_{2,-\pi} \\ \lambda_{3,-\pi} \\ \lambda_{4,-\pi} \end{pmatrix} A_4 e^{j\beta_\pi x} \tag{4}$$

where $(\lambda_{i,\pm r})$ are the eigenvectors associated with eigenvalues $\pm\beta_r$, and A_1, A_2, A_3, and A_4 are the amplitudes of launched modes, which can be estimated from the boundary conditions at $x = 0$ and $x = l$. However, considering the complicated expressions for the eigenvectors given by equations (3), this task seems to be difficult. Therefore, taking into account that the well-known backward-coupling couplers work purely only for $k_L = k_C = s$ ($d = 0$), we restrict further analysis to that single case. From (2) we obtain

$$\beta_c = \beta_0\sqrt{1\pm\delta} \qquad \beta_\pi = \beta_0\sqrt{1\pm\delta} \tag{5a}$$

where

$$\beta_0 = \sqrt{\frac{\beta_1^2+\beta_2^2}{2} - \beta_1\beta_2 s^2} \text{ and } \delta = \sqrt{1 - \frac{\beta_1^2\beta_2^2}{\beta_0^4}(1-s^2)^2}.$$

The sign in (5a) can be chosen quite arbitrarily, without any repercussions on the analysis. As will be shown in Section III, for the case where $k_L = k_C$ and in proximity to that point (for $k_L \simeq k_C$), two normal modes have the voltage vector in phase (and current vector in antiphase). This is in contrast to the situation which appears in commonly analyzed structures, e.g. coupled microstrips, where one of the two modes has voltage (and current) vectors in phase and is called even or c mode and the other has them in antiphase and is called odd or π mode. Let us assume that

$$\beta_c = \beta_0\sqrt{1+\delta} \text{ and } \beta_\pi = \beta_0\sqrt{1-\delta}. \tag{5b}$$

Then, the following identity can be proved:

$$\beta_2 + \beta_c = \beta_1 + \beta_\pi \qquad \text{for } \beta_1 > \beta_2$$

or

$$\beta_1 + \beta_c = \beta_2 + \beta_\pi \qquad \text{for } \beta_1 < \beta_2.$$

Thus it is evident that, for the nonhomogeneous lines when $\beta_1 \neq \beta_2$, the normal modes for asymmetric coupled lines in an inhomogeneous medium are propagating with different velocities.

The eigenvectors associated with $\pm\beta_c$ and $\pm\beta_\pi$ eigenvalues can be given (for the case when $k_L = k_C$) in the form

$$\begin{pmatrix} a_+ \\ b_+ \\ a_- \\ b_- \end{pmatrix}_{+c} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ q \end{pmatrix} \quad \begin{pmatrix} a_+ \\ b_+ \\ a_- \\ b_- \end{pmatrix}_{-c} = \begin{pmatrix} 0 \\ 1 \\ 1/q \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} a_+ \\ b_+ \\ a_- \\ b_- \end{pmatrix}_{+\pi} = \begin{pmatrix} 0 \\ 1 \\ q \\ 0 \end{pmatrix} \quad \begin{pmatrix} a_+ \\ b_+ \\ a_- \\ b_- \end{pmatrix}_{-\pi} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1/q \end{pmatrix} \tag{6}$$

where

$$q = \frac{\sqrt{\beta_1\beta_2}\,s}{\beta_c+\beta_2} = \frac{\beta_\pi-\beta_1}{\sqrt{\beta_1\beta_2}\,s}.$$

The form of these vectors clearly shows the mechanism of backward contradirectional coupling in which forward waves (+modes) can only couple to backward waves (−modes).

Since the eigenvectors have already been found, we can write two sets of equations by inserting $x = 0$ and $x = l$ into (4):

$$\begin{pmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{pmatrix} = (c)\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} \text{ and } \begin{pmatrix} b_3 \\ b_4 \\ a_3 \\ a_4 \end{pmatrix} = (c)\begin{pmatrix} A_1 e^{-j\theta_c} \\ A_2 e^{j\theta_c} \\ A_3 e^{-j\theta_\pi} \\ A_4 e^{j\theta_\pi} \end{pmatrix} \tag{7}$$

where (c) is a matrix whose columns are eigenvectors given by (6); θ_c and θ_π are electric lengths of coupled-line sections for c and π modes, respectively; and a_j and b_j are the incident and reflected waves at port j. From (7) we can easily obtain the transfer T matrix, relating the incident and reflected waves at the input (1,2) and output (3,4) ports:

$$\begin{pmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{pmatrix} = (T) \begin{pmatrix} b_3 \\ b_4 \\ a_3 \\ a_4 \end{pmatrix} \quad (8)$$

where $(T) = (c)(d)$ and (d) is a matrix obtained by inversion of the c matrix and by multiplying their rows by $e^{j\theta_c}$, $e^{-j\theta_c}$, $e^{j\theta_\pi}$, and $e^{-j\theta_\pi}$, respectively. The following formulas for the T parameters result:

$$T_{11} = \frac{q}{1-q^2}\left(\frac{1}{q}e^{j\theta_c} - qe^{-j\theta_\pi}\right) \qquad T_{12} = T_{13} = 0$$

$$T_{14} = -\frac{q}{1-q^2}\left(e^{j\theta_c} - e^{-j\theta_\pi}\right)$$

$$T_{21} = T_{24} = 0 \qquad T_{22} = -\frac{q}{1-q^2}\left(qe^{-j\theta_c} - \frac{1}{q}e^{j\theta_\pi}\right) \quad (9)$$

$$T_{23} = T_{14}^*$$

$$T_{31} = T_{34} = 0 \qquad T_{32} = -T_{23} \qquad T_{33} = T_{11}^*$$

$$T_{41} = -T_{14} \qquad T_{42} = T_{43} = 0 \qquad T_{44} = T_{22}^*.$$

Using relations between the S and T parameters, one can obtain

$$S_{11} = S_{22} = S_{33} = S_{44} = 0 \qquad S_{14} = S_{41} = S_{23} = S_{32} = 0$$

$$S_{12} = S_{21} = S_{34} = S_{43} = \frac{jK\sin\theta}{\sqrt{1-K^2}\cos\theta + j\sin\theta}$$

$$S_{13} = S_{31} = \frac{\sqrt{1-K^2}}{\sqrt{1-K^2}\cos\theta + j\sin\theta}e^{-j\theta u}$$

$$S_{24} = S_{42} = S_{13}e^{-j2\theta u} \quad (10a)$$

where

$$K = \frac{2q}{1+q^2} \qquad \theta = \frac{\theta_c + \theta_\pi}{2} \qquad u = \frac{\theta_c - \theta_\pi}{\theta_c + \theta_\pi}. \quad (10b)$$

The above results of analysis reveal the following properties of the structure of asymmetric coupled lines when they are "compensated" ($k_L = k_C$) and terminated characteristically ($Z_{iT} = Z_i$): 1) The structure composes an ideal "backward" contradirectional coupler at all frequencies. 2) The frequency dependences of the coupling and transmission losses in the main line are expressed by the same relations as for conventional couplers (in both symmetrical and asymmetrical arrangements of homogeneous coupled lines) [19], [17]; the difference consists only in replacing in the known formulas the coupling coefficient k with the effective coupling coefficient K, which is a function of s and a so-called asymmetry factor n, defined as β_2/β_1 for $\beta_1 > \beta_2$ or β_1/β_2 for $\beta_1 < \beta_2$. 3) As we can see, the phase shift between the output signals $\Delta\psi = \pi/2(1 + f/f_0 \cdot u)$, where f_0 and f are the center frequency (for $\theta = \pi/2$) and the sample frequency, respectively. Deviation of the phase shift from 90° is proportional to the frequency and to the $u(s,n)$ function which evaluates the relative difference between phase velocities of the c and π modes. The behavior of the $K(s,n)$ and $u(s,n)$ functions is illustrated by curves plotted in Fig. 2.

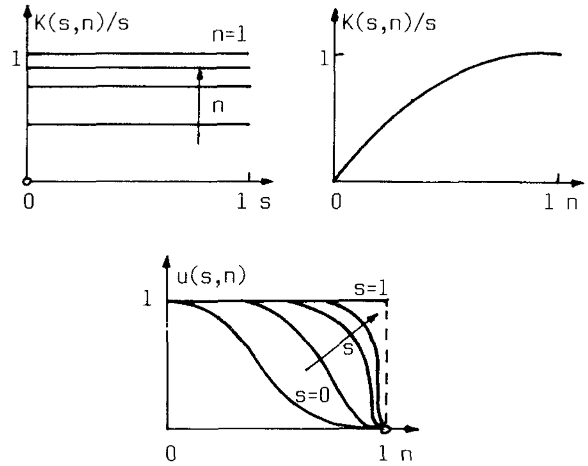

Fig. 2 Behavior of the $K(s,n)$ and $u(s,n)$ functions.

The conclusion that arises from the behavior of these functions is as follows: 1)The coupling decreases with an increase in asymmetry. For the case where phase constants β_1 and β_2 are substantially different ($n \approx 0$), the effective coupling of the lines is weak even if the coupling coefficient of these lines is high. 2) The higher the degree of asymmetry, the higher the deviation of phase shift from 90°. For tightly coupled lines a small difference in the phase constants can substantially perturb the phase response of the coupler.

In order to confirm the validity of the developed analysis, the S parameters as a function of the normalized frequency f/f_0 have been calculated, assuming the following coupled-mode parameters: $Z_1 = 50\ \Omega$, $Z_2 = 70\ \Omega$, $\beta_2/\beta_1 = 0.9$, and $s = 0.5$. These calculations have been carried out using two independent methods of analysis, namely the method applying coupled-mode formulation, which is the one presented here, and the method developed by Tripathi [2] that formulates the Z matrix, mentioned above in subsection A. In other words, the S parameters have been calculated using equations (10a) (the result of the first method of the analysis) and equations (1) (the result of the second). The effective coupling coefficient K and relative difference of the mode phase velocities u, computed according to equations (10b), are equal to 0.4993 and 0.06075, respectively. Next, the normal-mode parameters which are necessary for the second calculations have been estimated using expressions derived by Kal *et al.* [7, eqs. (4)–(6)] and appear as follows: $Z_{1c} = -Z_{1\pi} = 50\ \Omega = Z_1$, $Z_{2\pi} = -Z_{2c} = 70\ \Omega = Z_2$, $R_c = 0.316534532$, and $R_\pi = 4.42289816$. As has been stated from the second calculations (using (1)), $|S_{jj}|$, $|S_{14}|$, and $|S_{23}|$ are less than 10^{-8} at all frequencies. It has also been found that the frequency responses of coupling, transmission losses, and of phase shift agree exactly with the ones calculated using equations (10a).

The mode impedances that have been calculated and presented here reached unexpected, negative values. The singular behavior of the normal-mode parameters in proximity to the point when $k_L = k_C$ and at that point will be discussed in Section III. We shall present below the coupled-mode analysis extended to the case when $d \neq 0$ in order to determine the effect of nonideal equalization of the inductive and capacitive coupling coefficients on the coupler critical parameters.

C. Perturbation of the Coupler Critical Parameters ($d \neq 0$, $d/s \ll 1$)

Using general expressions for the eigenvectors given by equations (3), restricting the analysis only to the case where $d/s \ll 1$ in order to get simpler final formulas, and developing the coupled-mode analysis in the same fashion as was done previously, we obtain the following terms of the c and d matrices:

$$c_{11} = c_{22} = c_{23} = c_{14} = 1 \qquad c_{41} = c_{33} = q$$

$$c_{32} = c_{44} = \frac{1}{q}$$

$$c_{21} = qa_1(d/s) \qquad c_{31} = qb_1(d/s)$$

$$c_{12} = \frac{1}{q}a_2(d/s) \qquad c_{42} = \frac{1}{q}b_2(d/s)$$

$$c_{13} = qa_3(d/s) \qquad c_{43} = qb_3(d/s)$$

$$c_{24} = \frac{1}{q}a_4(d/s) \qquad c_{34} = \frac{1}{q}b_4(d/s) \tag{11}$$

where

$$a_1 = \left[(\beta_1+\beta_c)(\beta_2+\beta_c)+\beta_1\beta_2 s^2\right]/a$$

$$a_2 = \left[(\beta_1-\beta_c)(\beta_2-\beta_c)+\beta_1\beta_2 s^2\right]/a$$

$$a_3 = \left[(\beta_1-\beta_\pi)(\beta_2-\beta_\pi)+\beta_1\beta_2 s^2\right]/b$$

$$a_4 = \left[(\beta_1+\beta_\pi)(\beta_2+\beta_\pi)+\beta_1\beta_2 s^2\right]/b$$

$$b_1 = 2\sqrt{\beta_1\beta_2}\, s\beta_2/a \qquad b_2 = 2\sqrt{\beta_1\beta_2}\, s\beta_1/a$$

$$b_3 = 2\sqrt{\beta_1\beta_2}\, s\beta_1/b \qquad b_4 = 2\sqrt{\beta_1\beta_2}\, s\beta_2/b$$

$$a = (\beta_1+\beta_c)(\beta_1-\beta_c)-\beta_1\beta_2 s^2$$

$$b = (\beta_1+\beta_\pi)(\beta_1-\beta_\pi)-\beta_1\beta_2 s^2$$

and

$$d_{11} = pe^{j\theta_c} \qquad d_{41} = -pq^2e^{-j\theta_\pi} \qquad d_{22} = -pq^2e^{-j\theta_c}$$

$$d_{32} = pe^{j\theta_\pi}$$

$$d_{23} = -\frac{1}{q}d_{22} \qquad d_{33} = -qd_{32} \qquad d_{14} = -qd_{11}$$

$$d_{44} = -\frac{1}{q}d_{41}$$

$$d_{21} = pq^2\left(c_{21}+qc_{34}-q^2c_{24}-\frac{1}{q}c_{31}\right)e^{-j\theta_c}(d/s)$$

$$d_{31} = pq^2\left(-\frac{1}{q^2}c_{21}-qc_{34}+c_{24}+\frac{1}{q}c_{31}\right)e^{j\theta_\pi}(d/s)$$

$$d_{12} = pq^2\left(q^2c_{12}+\frac{1}{q}c_{43}-qc_{42}-c_{13}\right)e^{j\theta_c}(d/s)$$

$$d_{42} = pq^2\left(-\frac{1}{q^2}c_{12}-\frac{1}{q}c_{43}-c_{13}-qc_{42}\right)e^{-j\theta_\pi}(d/s)$$

$$d_{13} = pq^2\left(-\frac{1}{q}c_{12}-c_{43}-c_{42}+\frac{1}{q}c_{13}\right)e^{j\theta_c}(d/s)$$

$$d_{43} = pq^2(c_{43}+qc_{12}-qc_{13}-c_{42})e^{-j\theta_\pi}(d/s)$$

$$d_{24} = pq^2(-c_{34}-qc_{21}+c_{31}+qc_{24})e^{-j\theta_c}(d/s)$$

$$d_{34} = pq^2\left(c_{34}+\frac{1}{q}c_{21}-c_{31}-\frac{1}{q}c_{24}\right)e^{j\theta_\pi}(d/s) \tag{12}$$

where

$$p = \frac{1}{1-q^2}.$$

By multiplication of these two matrices, the transfer T matrix can be obtained and used for calculating the coupler parameters. It has been proved that coupler matching, directivity, and phase shift are the only ones to be perturbated substantially when the relative difference between coupling coefficients, d/s, increases. The coupler directivity, for instance, can be computed from the following relation:

$$D(\text{dB}) = 20\log(L/M)+20\log|s/d| \tag{13}$$

where

$$L = \frac{2}{K}\sin\theta\sqrt{1-K^2\cos^2\theta}$$

$$M = \sqrt{(S\sin\theta u\cos\theta + T\cos\theta u\sin\theta)^2+(V\sin\theta u\sin\theta)^2}$$

$$S = \left(q-\frac{1}{q}\right)a_1+(1-q^2)b_4 \qquad T = q(a_1+a_4)-b_1-b_4$$

$$V = \frac{1}{q}a_1+qa_4+q^2b_4-b_1.$$

Similar relationships can be derived for calculating the reflection coefficients $|S_{11}|$ and $|S_{22}|$.

The two examples of numerical calculations illustrate the usefulness of the above analysis. In the first, we have

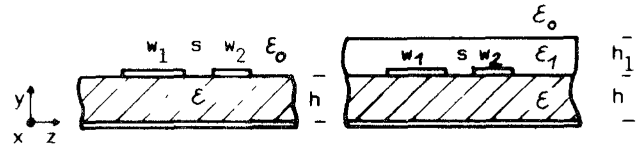

Fig. 3. Cross-sectional view of coupled microstrip lines (a) without and (b) with dielectric overlay

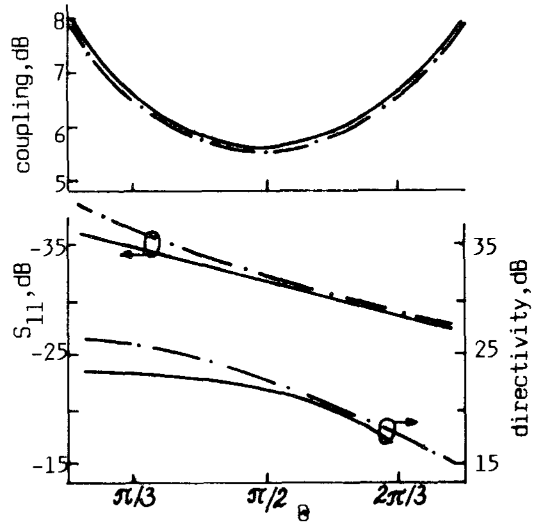

Fig. 4. Coupling, reflection coefficient $|S_{11}|$, and directivity against normalized frequency θ for a coupler with Fig. 3(a) configuration of coupled lines ($w_1/h = 0.4$, $w_2/h = 0.11$, $s/h = 0.08$, $\epsilon_r = 10$) —— [12], optimized terminations $Z_{1T} = 60.7\ \Omega$ and $Z_{2T} = 85.0\ \Omega$; — · — this theory, characteristic terminations $Z_{1T} = Z_1 = \sqrt{L_1/C_1} = 61.0\ \Omega$ and $Z_{2T} = Z_2 = \sqrt{L_2/C_2} = 84.6\ \Omega$.

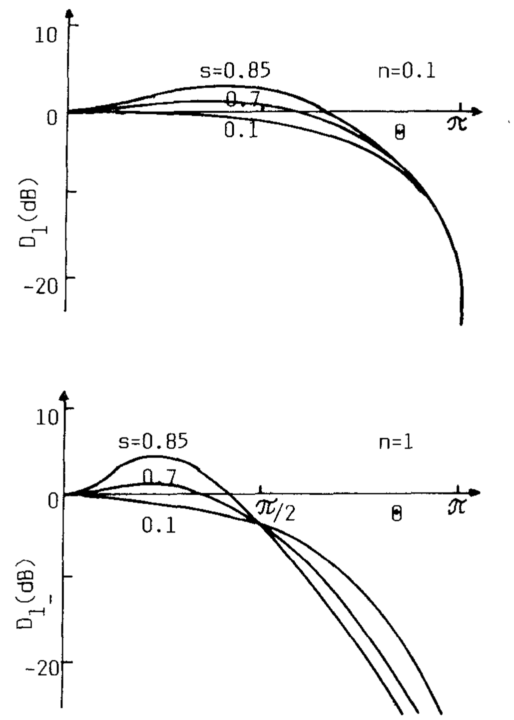

Fig. 5. Coupler directivity $D(\mathrm{dB}) = D_1(\mathrm{dB}) + 20\log|s/d|$ against normalized frequency $\theta = (\pi/2)f/f_0$ with n and s coefficients as parameters.

estimated the frequency-dependent parameters of the coupler designed by Tripathi [12], who applied the n.m.c. termination concept, the Z–S matrix transformation, and an iterative procedure which can optimize the coupler performance in terms of the terminating impedances. The structural parameters of the asymmetric coupled microstrip lines shown in Fig. 3(a) have been chosen to be the same as those of Tripathi ($w_1/h = 0.4$, $w_2/h = 0.11$, $s/h = 0.08$, and $\epsilon_r = 10$).

The results of our calculations carried out using the spectral-domain approach and variational method [20] are $L_1/\mu_0 = 0.4683$, $L_2/\mu_0 = 0.6376$, $L_m/\mu_0 = 0.3015$, $C_1/\epsilon_0 = 17.88$, $C_2/\epsilon_0 = 12.66$, and $C_m/\epsilon_0 = 7.548$. From these self- and mutual inductances and capacitances the following coupled-mode parameters have been estimated: $k_L = 0.5518$, $k_C = 0.5017$, $\beta_2/\beta_1 = 0.9817$, $Z_1 = 61.0\ \Omega$, and $Z_2 = 84.6\ \Omega$. The characteristics computed according to equations (10)–(13) can be compared to the ones derived by Tripathi [12] (see Fig. 4). It should be pointed out that the terminating impedances obtained from Cristal's approximate TEM approach (assumed by Tripathi as a suitable starting point for the optimization procedure) give a rather poor matching (for $Z_{1T} = \sqrt{Z_{1c}Z_{1\pi}} = 49.9\ \Omega$ and $Z_{2T} = \sqrt{Z_{2c}Z_{2\pi}} = 103.2\ \Omega$, $|S_{11}|$ is even less than -15 dB). Though the optimization procedure used by Tripathi made it possible to minimize the reflection coefficient at the input ports, the transmission coefficient between the input and the isolated port remained high. The reason is that the parameters of the coupler designed utilizing both the n.m.c. termination concept and the optimization procedure depend on the difference between the normal-mode phase velocities (in that case $\Delta v/v \simeq 0.0363$). The almost ideal realization of that coupler achieved in the overlayered microstrip structure (shown in Fig. 3(b)) will be presented in Section III.

In the second example, mentioned before, we present numerical calculations of the frequency-dependent directivity of the coupler carried out using equations (13) for the symmetrical version ($n = 1$) of the coupler as well as the asymmetrical one with a high degree of asymmetry ($n = 0.1$). The results of the calculations are plotted in Fig. 5. An excellent agreement with the results of similar calculations carried out using Krage and Haddad's expressions [19], derived for the symmetrical structure of coupled lines, has been stated. Generally, as can be seen from Fig. 5, the directivity of the asymmetrical coupled-line coupler is higher than that of the symmetrical counterpart.

III. Design of a Coupler with Improved Directivity and Behavior of Normal-Mode Parameters

To design a very high directivity coupler, it is necessary to apply the structure of coupled lines for which the equalization of the coupling coefficients might be achieved. Such a structure is expected to be a coupled line structure with a dielectric overlay (shown in Fig. 3(b)). In order to

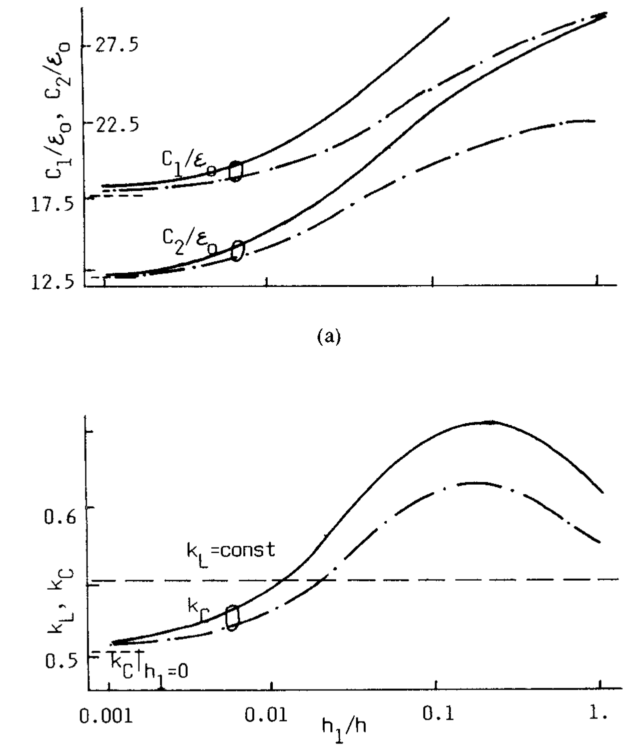

Fig. 6. (a) Self-capacitances C_1 and C_2 and (b) coupling coefficients k_L and k_C of coupled lines from Fig. 3(b) versus h_1/h for $\epsilon_{r1}=16$ (——) and 10 (—·—). The structural parameters of coupled lines are the same as those given in Fig. 4.

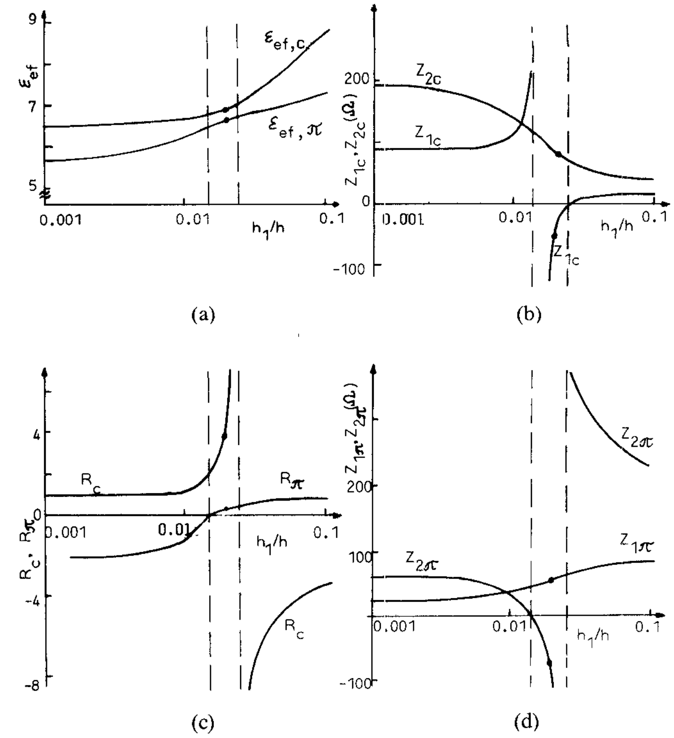

Fig. 7. (a) Effective dielectric constants $\epsilon_{ef,r}=\{\beta_r/(\omega\sqrt{\mu_0\epsilon_0})\}^2$, $r=c,\pi$. (b) Voltage mode numbers R_r. (c), (d) Modal characteristic impedances Z_{ir}, $i=1,2$, of coupled lines with Fig. 3(b) configuration versus h_1/h. $\epsilon_{r1}=\epsilon_r$. The structural parameters of coupled lines are given in Fig. 4.

prove its usefulness in a high-directivity asymmetric coupler design, the coupled line parameters have been calculated versus the dielectric overlayer thickness h_1 for several values of the dielectric constant of this layer ϵ_{r1}. The same structural parameters as those of Tripathi and the ones of our previous calculations have been chosen. The self-inductances and the mutual inductance per unit length of the lines have already been calculated, as they are independent from the dielectric filling.

For the structure without the overlay, $k_C<k_L$. Initially, we might expect that as the thickness of the overlay with the same dielectric constant as that of the main substrate is increased, k_C should approach k_L and, when the entire structure is filled, k_C should be equal to k_L. Therefore, only dielectric layers with higher dielectric constants should be used to achieve the required equalization for a given thickness of the dielectric overlay. However, this is not true, as the results prove (see Fig. 6(b)). In both parts of Fig. 6, curves have been plotted which show C_1, C_2, and k_C versus the thickness of the overlay. For $\epsilon_{r1}=\epsilon_r=10$, the equalization can be reached for $h_1/h \simeq 0.02$. As has been calculated, the directivity of this coupler (with $d/s=4.9\cdot10^{-4}$ for $h_1/h=0.02$) is better than 60 dB, and the return loss is better than -60 dB over a frequency of one octave. These parameters can be reached under the assumption that the dispersion of the coupled lines can be neglected and the coupler terminating impedances Z_{1T} and Z_{2T} are equal to the characteristic impedances $Z_1=55.4\ \Omega$ and $Z_2=77.3\ \Omega$, respectively. The calculations of the frequency-dependent characteristics of the asymmetric coupled lines which have been carried out using the spectral-domain full-wave analysis (not yet published) make it possible to state that the dispersion effect on the coupler parameters can be neglected up to the normalized frequency $h/\lambda=0.01$.

It is interesting to examine the behavior of the normal-mode parameters in the proximity of the point where, for the considered lines, $k_L=k_C$ ($d=0$). These normal-mode parameters have been calculated introducing the coupled line unit parameters into the expressions derived by Tripathi [2, eqs. (6), (7), (13)–(16)]. The computed characteristics of the propagation constants β_c and β_π, voltage mode numbers R_c and R_π, and mode impedances Z_{ic} and $Z_{i\pi}$ have been plotted versus the normalized dielectric overlay thickness h_1/h (Fig. 7).

In the curves of Fig. 7, the critical point of $k_L=k_C$ has been marked. For that point and in close proximity to it, the propagation constants are notably modified, because of significant changes in the field distribution of the two modes. Moreover, it is for that point that the expressions for Z_{ic} and $Z_{i\pi}$ (given e.g. by eq. (6) in [11]) become indeterminate. As can be proved, however, by computing limiting values for those quantities as $d\to0$, the following relations are obtained:

$$Z_{1c}=\pm Z_1 \qquad Z_{2c}=\mp Z_2 \qquad Z_{1\pi}=\mp Z_1 \quad \text{and } Z_{2\pi}=\pm Z_2 \tag{14}$$

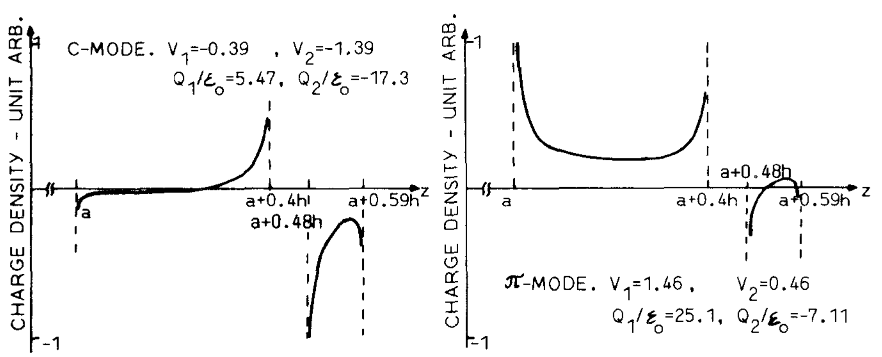

Fig. 8 Sketch of the charge surface density distributed on the two strips of analyzed system of coupled lines from Fig. 3(b) for the two modes in the case of $k_L \simeq k_C$. V_i and Q_i/ϵ_0 are the potential and total charge on the strip i, respectively. $h_1/h = 0.02$, $\epsilon_{r1} = \epsilon_r = 10$.

where the upper or lower signs are adequate for the cases where $\beta_2 < \beta_1$ and $\beta_2 > \beta_1$, respectively. These relations were discussed above in Section II. Equations (14) together with the known relations between the normal-mode parameters lead to the following relation:

$$R_c R_\pi = \frac{Z_2}{Z_1}. \tag{15}$$

From examining the expressions for the mode impedances (see [7, eqs. (5) and (6)]), it is seen that there are two points (the first when $k_C = \beta_1/\beta_2 k_L$ and the second when $k_C = \beta_2/\beta_1 k_L$) for which either the $Z_{1c(\pi)}$ or the $Z_{2\pi(c)}$ is zero (the ones that assume negative values for the critical point and in its proximity). For the points $k_C = \beta_1/\beta_2 k_L$ and $k_C = \beta_2/\beta_1 k_L$, the voltage mode numbers R_c and R_π are equal to zero or infinity, as can be observed from examining the expressions given in [7, eqs. (4)]. An infinite value of the voltage mode ratio for the $c(\pi)$ mode is associated with a zero value of the current mode ratio for the $\pi(c)$ mode. Thus, this is the situation predicted by Kajfez [21], in which there is a unilateral power flow (one conductor of a pair of transmission lines carries the power in the $+x$ direction while the other has either a zero voltage or a zero current). We can also discuss the situation in which power flows antiparallel (when one conductor carries the power in the $+x$ direction and the other in the $-x$ direction). This happens for the critical point $k_L = k_C$ and in its proximity when two normal modes have the voltage vector in phase and the current vector in antiphase, and appropriate mode impedances have negative values. These remarks can complete Kajfez's considerations [21], where questions concerning the applications of the lines with such properties, although formulated, are left with no answer.

In Fig. 8, a surface density of the charge distributed on the two strips of the analyzed system of coupled lines, computed for the two modes in the case where $k_L \simeq k_C$, is plotted. These curves are expected to be helpful in explaining the phenomena associated with this type of propagation of the normal modes. As has been already mentioned, these independent normal modes cannot be physically excited separately by a system of voltage sources and terminating impedances, because the condition for the n.m.c. terminations [11], namely $Z_{2T}/Z_{1T} = -R_c R_\pi$, cannot be satisfied in the circumstances where $R_c R_\pi > 0$.

Finally, notice the following simple, easily derived relation between the q term of eigenvectors given by equations (6) and the voltage mode numbers R_c and R_π:

$$q = \sqrt{\frac{R_c}{R_\pi}}. \tag{16}$$

The effective coupling coefficient K can then be expressed as

$$K = \frac{2\sqrt{R_c R_\pi}}{R_c + R_\pi}. \tag{17}$$

Equations (14)–(17) together with equations (10) are expected to be especially useful in designing couplers for higher frequencies, by using the full-wave analysis and normal-mode parameters rather than the quasi-static analysis and coupled-mode parameters.

IV. Conclusion

Applying the coupled-mode formulation of inhomogeneous lines, it has been proved that the structure of inhomogeneous asymmetric coupled lines can provide an ideal backward-coupling directional coupler, if the inductive k_L and capacitive k_C coupling coefficients are equal and the terminating impedances Z_{iT} are equal to the characteristic impedances $Z_i (i = 1,2)$ of uncoupled lines. The phase velocities of two normal modes propagating in this structure under the assumption that the coupling coefficients are equal can be substantially different. This difference can be determined by the simple relation $\beta_c - \beta_\pi = \pm(\beta_1 - \beta_2)$. The behavior of the new ideal asymmetrical coupler, in contrast to all well-known ideal couplers, cannot be investigated in the context of the n.m.c. termination concept.

In the literature on the backward-coupling directional couplers, many authors [10]–[12], [22] refer to the statement deduced by Louisell [23], who ascertained the existence of the ideal coupler under the condition where the waves are synchronous ($\beta_1 = \beta_2$). This condition, together with the assumption that the coupling coefficients are equal, implies further that only the symmetrical coupled-line coupler in which two normal modes can be guided with the same velocities can be ideally realized. The S matrix of the ideal, asymmetrical coupler has been derived and perturbation of the critical coupler parameters has been investigated. The behavior of the normal-mode parameters (the voltage mode numbers and modal characteristic impedances) has been examined in proximity to the critical point, when $k_L = k_C$, and at that point. For that point and in close proximity to it, the propagation constants are notably modified, because of certain significant changes in the field distribution of the two modes. It has also been stated that in proximity to that critical point the power carried by a pair of transmission lines flows antiparallel. The numerical results presented here confirm the validity of the developed analysis and prove the possibility of a very high directivity asymmetrical coupler design.

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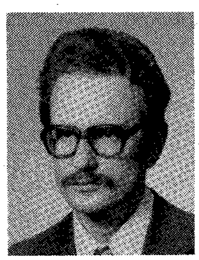

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